Waldemar Pompe

Korn's First Inequality with variable coefficients and its generalization

Comment.Math.Univ.Carolinae 44,1 (2003) 57-70.

Abstract: If $\Omega \subset \mathbb{R}^n$ is a bounded domain with Lipschitz boundary $\partial\Omega$ and Γ is an open subset of $\partial\Omega$, we prove that the following inequality

$$\left(\int_\Omega |A(x)\nabla u(x)|^p \, dx\right)^{1/p} + \left(\int_\Gamma |u(x)|^p \, d\mathcal{H}^{n-1}(x)\right)^{1/p} \geq c \, \|u\|_{W^{1,p}(\Omega)}$$

holds for all $u \in W^{1,p}(\Omega;\mathbb{R}^m)$ and $1 < p < \infty$, where

$$(A(x)\nabla u(x))_k = \sum_{i=1}^{m} \sum_{j=1}^{n} a_k^{ij}(x) \, \frac{\partial u_i}{\partial x_j}(x) \quad (k = 1, 2, \ldots, r; \; r \geq m)$$

defines an elliptic differential operator of first order with continuous coefficients on $\overline{\Omega}$. As a special case we obtain

$$(*) \qquad \int_\Omega \left|\nabla u(x)F(x) + (\nabla u(x)F(x))^T\right|^p dx \geq c \int_\Omega |\nabla u(x)|^p \, dx \, ,$$

for all $u \in W^{1,p}(\Omega;\mathbb{R}^n)$ vanishing on Γ, where $F : \overline{\Omega} \to M^{n\times n}(\mathbb{R})$ is a continuous mapping with $\det F(x) \geq \mu > 0$. Next we show that $(*)$ is not valid if $n \geq 3$, $F \in L^\infty(\Omega)$ and $\det F(x) = 1$, but does hold if $p = 2$, $\Gamma = \partial\Omega$ and $F(x)$ is symmetric and positive definite in Ω.

Keywords: Korn's Inequality, coercive inequalities
AMS Subject Classification: 35F15, 35J55