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Filling boxes densely and disjointly

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Abstract: We effectively construct in the Hilbert cube $\mathbb{H} = [0,1]^\omega$ two sets $V, W \subset \mathbb{H}$ with the following properties:

(a) $V \cap W = \emptyset$,
(b) $V \cup W$ is discrete-dense, i.e. dense in ${[0,1]_D}^\omega$, where $[0,1]_D$ denotes the unit interval equipped with the discrete topology,
(c) V,W are open in $\mathbb{H}$. In fact, $V = \bigcup_\mathbb{N} V_i$, $W = \bigcup_\mathbb{N} W_i$, where $V_i = \bigcup_0^{2^{i-1}-1} V_{ij}$, $W_i = \bigcup_0^{2^{i-1}-1} W_{ij}$. V_{ij}, W_{ij} are basic open sets and $(0,0,0,\dots) \in V_{ij}$, $(1,1,1,\dots) \in W_{ij}$,
(d) $V_i \cup W_i$, $i \in \mathbb{N}$ is point symmetric about $(1/2, 1/2, 1/2, \dots)$.

Instead of $[0,1]$ we could have taken any T_4-space or a digital interval, where the resolution (number of points) increases with i.

Keywords: Hilbert cube, discrete-dense, disjoint, disconnected, covering, constructive, computation, digital interval, T_4-space
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