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***Cardinal characteristics of the ideal of Haar null sets***

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**Abstract:** We calculate the cardinal characteristics of the $\sigma$-ideal $\mathcal{HN}(G)$ of Haar null subsets of a Polish non-locally compact group $G$ with invariant metric and show that $\mathrm{cov}(\mathcal{HN}(G)) \le \mathfrak{b} \le \max\{\mathfrak{d}, \mathrm{non}(\mathcal{N})\} \le \mathrm{non}(\mathcal{HN}(G)) \le \mathrm{cof}(\mathcal{HN}(G)) > \min\{\mathfrak{d}, \mathrm{non}(\mathcal{N})\}$. If $G = \prod_{n\ge 0} G_n$ is the product of abelian locally compact groups $G_n$, then $\mathrm{add}(\mathcal{HN}(G)) = \mathrm{add}(\mathcal{N})$, $\mathrm{cov}(\mathcal{HN}(G)) = \min\{\mathfrak{b}, \mathrm{cov}(\mathcal{N})\}$, $\mathrm{non}(\mathcal{HN}(G)) = \max\{\mathfrak{d}, \mathrm{non}(\mathcal{N})\}$ and $\mathrm{cof}(\mathcal{HN}(G)) \ge \mathrm{cof}(\mathcal{N})$, where $\mathcal{N}$ is the ideal of Lebesgue null subsets on the real line. Martin Axiom implies that $\mathrm{cof}(\mathcal{HN}(G)) > 2^{\aleph_0}$ and hence $G$ contains a Haar null subset that cannot be enlarged to a Borel or projective Haar null subset of $G$. This gives a negative (consistent) answer to a question of S. Solecki. To obtain these estimates we show that for a Polish non-locally compact group $G$ with invariant metric the ideal $\mathcal{HN}(G)$ contains all $o$-bounded subsets (equivalently, subsets with the small ball property) of $G$.

**Keywords:** Polish group, Haar null set, Martin Axion, cardinal characteristics of an ideal, $o$-bounded set, the small ball property
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