Abraham A. Ungar

The hyperbolic triangle centroid

Comment.Math.Univ.Carolinae 45,2 (2004) 355-369.

Abstract: Some gyrocommutative gyrogroups, also known as Bruck loops or K-loops, admit scalar multiplication, turning themselves into gyrovector spaces. The latter, in turn, form the setting for hyperbolic geometry just as vector spaces form the setting for Euclidean geometry. In classical mechanics the centroid of a triangle in velocity space is the velocity of the center of momentum of three massive objects with equal masses located at the triangle vertices. Employing gyrovector space techniques we find in this article that, in full analogy, the centroid of a hyperbolic triangle in relativity velocity space is the velocity of the center of momentum of three massive objects with equal rest masses located at the triangle vertices. Being guided by the relativistic mass correction of moving massive objects in special relativity theory, we express the hyperbolic triangle centroid in terms of the triangle vertices, resulting in a novel hyperbolic triangle centroid identity that captures remarkable analogies with its Euclidean counterpart.

Keywords: loops, gyrogroups, gyrovector spaces, hyperbolic geometry, Einstein addition, Möbius transformation
AMS Subject Classification: 20N05, 51P05, 83A05