

Brian Jefferies, Susumu Okada
Semivariation in L^p -spaces

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Abstract: Suppose that X and Y are Banach spaces and that the Banach space $X \hat{\otimes}_\tau Y$ is their complete tensor product with respect to some tensor product topology τ . A uniformly bounded X -valued function need not be integrable in $X \hat{\otimes}_\tau Y$ with respect to a Y -valued measure, unless, say, X and Y are Hilbert spaces and τ is the Hilbert space tensor product topology, in which case Grothendieck's theorem may be applied.

In this paper, we take an index $1 \leq p < \infty$ and suppose that X and Y are L^p -spaces with τ_p the associated L^p -tensor product topology. An application of Orlicz's lemma shows that not all uniformly bounded X -valued functions are integrable in $X \hat{\otimes}_{\tau_p} Y$ with respect to a Y -valued measure in the case $1 \leq p < 2$. For $2 < p < \infty$, the negative result is equivalent to the fact that not all continuous linear maps from ℓ^1 to ℓ^p are p -summing, which follows from a result of S. Kwapien.

Keywords: absolutely p -summing, bilinear integration, semivariation, tensor product

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