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Some relative properties on normality and paracompactness, and their absolute embeddings

Abstract: Paracompactness (= 2-paracompactness) and normality of a subspace Y in a space X defined by Arhangel'skii and Genedi [4] are fundamental in the study of relative topological properties ([2], [3]). These notions have been investigated by primary using of the notion of weak C- or weak P-embeddings, which are extension properties of functions defined in [2] or [18]. In fact, Bella and Yaschenko [8] characterized Tychonoff spaces which are normal in every larger Tychonoff space, and this result is essentially implied by their previous result in [8] on a corresponding case of weak C-embeddings. In this paper, we introduce notions of 1-normality and 1-collectionwise normality of a subspace Y in a space X, which are closely related to 1-paracompactness of Y in X. Furthermore, notions of quasi-C^*- and quasi-P-embeddings are newly defined. Concerning the result of Bella and Yaschenko above, by characterizing absolute cases of quasi-C^*- and quasi-P-embeddings, we obtain the following result: a Tychonoff space Y is 1-normal (or equivalently, 1-collectionwise normal) in every larger Tychonoff space if and only if Y is normal and almost compact. As another concern, we also prove that a Tychonoff (respectively, regular, Hausdorff) space Y is 1-metacompact in every larger Tychonoff (respectively, regular, Hausdorff) space if and only if Y is compact. Finally, we construct a Tychonoff space X and a subspace Y such that Y is 1-paracompact in X but not 1-subparacompact in X. This is a negative answer to a question of Qu and Yasui in [25].

Keywords: 1-paracompactness of Y in X, 2-paracompactness of Y in X, 1-collectionwise normality of Y in X, 2-collectionwise normality of Y in X, 1-normality of Y in X, 2-normality of Y in X, quasi-P-embedding, quasi-C-embedding, quasi-C^*-embedding, 1-metacompactness of Y in X, 1-subparacompactness of Y in X
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