Emad Abu Osba, Melvin Henriksen, Osama Alkam, F.A. Smith
The maximal regular ideal of some commutative rings

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Abstract: In 1950 in volume 1 of Proc. Amer. Math. Soc., B. Brown and N. McCoy showed that every (not necessarily commutative) ring R has an ideal $\mathfrak{M}(R)$ consisting of elements a for which there is an x such that $axa = a$, and maximal with respect to this property. Considering only the case when R is commutative and has an identity element, it is often not easy to determine when $\mathfrak{M}(R)$ is not just the zero ideal. We determine when this happens in a number of cases: Namely when at least one of a or $1 - a$ has a von Neumann inverse, when R is a product of local rings (e.g., when R is $\mathbb{Z}_n$ or $\mathbb{Z}_n[i]$), when R is a polynomial or a power series ring, and when R is the ring of all real-valued continuous functions on a topological space.

Keywords: commutative rings, von Neumann regular rings, von Neumann local rings, Gelfand rings, polynomial rings, power series rings, rings of Gaussian integers (mod n), prime and maximal ideals, maximal regular ideals, pure ideals, quadratic residues, Stone-Čech compactification, $C(X)$, zerosets, cozerosets, P-spaces
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