A.V. Arhangel'skii G_{δ} -modification of compacta and cardinal invariants

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Abstract: Given a space X, its G_{δ} -subsets form a basis of a new space X_{ω} , called the G_{δ} -modification of X. We study how the assumption that the G_{δ} -modification X_{ω} is homogeneous influences properties of X. If X is first countable, then X_{ω} is discrete and, hence, homogeneous. Thus, X_{ω} is much more often homogeneous than X itself. We prove that if X is a compact Hausdorff space of countable tightness such that the G_{δ} -modification of X is homogeneous, then the weight w(X) of X does not exceed 2^{ω} (Theorem 1). We also establish that if a compact Hausdorff space of countable tightness is covered by a family of G_{δ} -subspaces of the weight $\leq c = 2^{\omega}$, then the weight of X is not greater than 2^{ω} (Theorem 4). Several other related results are obtained, a few new open questions are formulated. Fedorchuk's hereditarily separable compactum of the cardinality greater than $c = 2^{\omega}$ is shown to be G_{δ} -homogeneous under CH. Of course, it is not homogeneous when given its own topology.

Keywords: weight, tightness, G_{δ} -modification, character, Lindelöf degree, homogeneous space

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