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Supremum properties of Galois-type connections

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Abstract: In a former paper, motivated by a recent theory of relators (families of relations), we have investigated increasingly regular and normal functions of one preordered set into another instead of Galois connections and residuated mappings of partially ordered sets.

A function f of one preordered set X into another Y has been called

(1) increasingly g -normal, for some function g of Y into X , if for any $x \in X$ and $y \in Y$ we have $f(x) \leq y$ if and only if $x \leq g(y)$;

(2) increasingly φ -regular, for some function φ of X into itself, if for any $x_1, x_2 \in X$ we have $x_1 \leq \varphi(x_2)$ if and only if $f(x_1) \leq f(x_2)$.

In the present paper, we shall prove that if f is an increasingly regular function of X onto Y , or f is an increasingly normal function of X into Y , then $f[\text{sup}(A)] \subset \text{sup}(f[A])$ for all $A \subset X$. Moreover, we shall also prove some more delicate, but less important supremum properties of such functions.

Keywords: preordered sets, Galois connections (residuated mappings), supremum properties

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