Ángel Tamariz-Mascarúa
Continuous selections on spaces of continuous functions

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Abstract: For a space Z, we denote by $\mathcal{F}(Z)$, $\mathcal{K}(Z)$ and $\mathcal{F}_2(Z)$ the hyperspaces of non-empty closed, compact, and subsets of cardinality ≤ 2 of Z, respectively, with their Vietoris topology. For spaces X and E, $C_p(X,E)$ is the space of continuous functions from X to E with its pointwise convergence topology.
We analyze in this article when $\mathcal{F}(Z)$, $\mathcal{K}(Z)$ and $\mathcal{F}_2(Z)$ have continuous selections for a space Z of the form $C_p(X,E)$, where X is zero-dimensional and E is a strongly zero-dimensional metrizable space. We prove that $C_p(X,E)$ is weakly orderable if and only if X is separable. Moreover, we obtain that the separability of X, the existence of a continuous selection for $\mathcal{K}(C_p(X,E))$, the existence of a continuous selection for $\mathcal{F}_2(C_p(X,E))$ and the weak orderability of $C_p(X,E)$ are equivalent when X is $\mathbb{N}$-compact.
Also, we decide in which cases $C_p(X,2)$ and $\beta C_p(X,2)$ are linearly orderable, and when $\beta C_p(X,2)$ is a dyadic space.

Keywords: continuous selections, Vietoris topology, linearly orderable space, weakly orderable space, space of continuous functions, dyadic spaces
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