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Some versions of relative paracompactness and their absolute embeddings

Abstract: Arhangel'skii [Sci. Math. Jpn. 55 (2002), 153–201] defined notions of relative paracompactness in terms of locally finite open partial refinement and asked if one can generalize the notions above to the well known Michael's criteria of paracompactness in [17] and [18]. In this paper, we consider some versions of relative paracompactness defined by locally finite (not necessarily open) partial refinement or locally finite closed partial refinement, and also consider closure-preserving cases, such as 1-lf-, 1-cp-, α-lf, α-cp-paracompactness and so on. Moreover, on their absolute embeddings, we have the following results. Theorem 1. A Tychonoff space Y is 1-lf- (or equivalently, 1-cp-) paracompact in every larger Tychonoff space if and only if Y is Lindelöf. Theorem 2. A Tychonoff space Y is α-lf- (or equivalently, α-cp-) paracompact in every larger Tychonoff space if and only if Y is compact. We also show that in Theorem 1, "every larger Tychonoff space" can be replaced by "every larger Tychonoff space containing Y as a closed subspace". But, this replacement is not available for Theorem 2.