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Weak-bases and D-spaces

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Abstract: It is shown that certain weak-base structures on a topological space give a D-space. This solves the question by A.V. Arhangel'skii of when quotient images of metric spaces are D-spaces. A related result about symmetrizable spaces also answers a question of Arhangel'skii.

Theorem. Any symmetrizable space X is a D-space (hereditarily).

Hence, quotient mappings, with compact fibers, from metric spaces have a D-space image. What about quotient s-mappings? Arhangel'skii and Buzyakova have shown that spaces with a point-countable base are D-spaces so open s-images of metric spaces are already known to be D-spaces.
A collection $\mathcal{W}$ of subsets of a sequential space X is said to be a w-system for the topology if whenever $x \in U \subseteq X$, with U open, there exists a subcollection $\mathcal{V} \subseteq \mathcal{W}$ such that $x \in \bigcap \mathcal{V}$, $\bigcup \mathcal{V}$ is a weak-neighborhood of x, and $\bigcup \mathcal{V} \subseteq U$.

Theorem. A sequential space X with a point-countable w-system is a D-space.

Corollary. A space X with a point-countable weak-base is a D-space.

Corollary. Any T_2 quotient s-image of a metric space is a D-space.

Keywords: quotient map, symmetrizable space, weak-base, w-structure, D-space
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