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Radical decompositions of semiheaps

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Abstract: Semiheaps are ternary generalisations of involuted semigroups. The first kind of semiheaps studied were heaps, which correspond closely to groups. We apply the radical theory of varieties of idempotent algebras to varieties of idempotent semiheaps. The class of heaps is shown to be a radical class, as are two larger classes having no involuted semigroup counterparts. Radical decompositions of various classes of idempotent semiheaps are given. The results are applied to involuted I-semigroups, leading to a radical-theoretic interpretation of the largest idempotent-separating congruence.

Keywords: radical theory of idempotent algebras, ternary operation, involuted semigroups, semiheaps, generalised heaps, heaps

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