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Hausdorff and packing dimensions for ergodic invariant measures of two-dimensional Lorenz transformations

Comment.Math.Univ.Carolin. 50,2 (2009) 221 –243.

Abstract: We extend the notions of Hausdorff and packing dimension introducing weights in their definition. These dimensions are computed for ergodic invariant probability measures of two-dimensional Lorenz transformations, which are transformations of the type occuring as first return maps to a certain cross section for the Lorenz differential equation. We give a formula of the dimensions of such measures in terms of entropy and Lyapunov exponents. This is done for two choices of the weights using the recurrence time of a set and equilibrium states respectively.

Keywords: Hausdorff dimension, packing dimension, Lorenz transformation, ergodic measure
AMS Subject Classification: 37D50, 28A78, 37C45, 37A35

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