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Lattices of Scott-closed sets

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Abstract: A dcpo P is continuous if and only if the lattice $C(P)$ of all Scott-closed subsets of P is completely distributive. However, in the case where P is a non-continuous dcpo, little is known about the order structure of $C(P)$. In this paper, we study the order-theoretic properties of $C(P)$ for general dcpo's P. The main results are: (i) every $C(P)$ is C-continuous; (ii) a complete lattice L is isomorphic to $C(P)$ for a complete semilattice P if and only if L is weak-stably C-algebraic; (iii) for any two complete semilattices P and Q, P and Q are isomorphic if and only if $C(P)$ and $C(Q)$ are isomorphic. In addition, we extend the function $P \mapsto C(P)$ to a left adjoint functor from the category **DCPO** of dcpo's to the category **CPAlg** of C-prealgebraic lattices.

Keywords: domain, complete semilattice, Scott-closed set, C-continuous lattice, C-algebraic lattice
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