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On the sign of Colombeau functions and applications to conservation laws

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Abstract: A generalized concept of sign is introduced in the context of Colombeau algebras. It extends the sign of the point-value in the case of sufficiently regular functions. This concept of generalized sign is then used to characterize the entropy condition for discontinuous solutions of scalar conservation laws.

Keywords: Colombeau algebra, generalized sign, conservation law, entropy condition
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References

[1] Colombeau J.-F., *Multiplication of distributions*, Bull. Amer. Math. Soc. (N.S.) **23** (1990), no. 2, 251–268.

[2] Colombeau J.-F., *Elementary introduction to new generalized functions*, North-Holland Mathematics Studies 113, Notes on Pure Mathematics 103, North-Holland Publishing Co., Amsterdam, 1985.

[3] Dafermos C.M., *Hyperbolic conservation laws in continuum physics*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences] 325, Springer, Berlin, 2000.

[4] Danilov V.G., Omel'yanov G.A., *Calculation of the singularity dynamics for quadratic nonlinear hyperbolic equations. Example: the Hopf equation*, Nonlinear Theory of Generalized Functions (Vienna, 1997), Chapman & Hall/CRC Res. Notes Math. 401, Chapman & Hall/CRC, Boca Raton, FL, 1999, 63–74.

[5] DiPerna R.J., Lions P.-L., *Ordinary differential equations, transport theory and Sobolev spaces*, Invent. Math. **98** (1989), no. 3, 511–547.

[6] Lions P.-L., Perthame B., Tadmor E., *A kinetic formulation of multidimensional scalar conservation laws and related equations*, J. Amer. Math. Soc. **7** (1994), no. 1, 169–191.

[7] Lojasiewicz S., *Sur la valeur et la limite d'une distribution en un point*, Studia Math. **16** (1957), 1–36.

[8] Nozari K., Afrouzi G.A., *Travelling wave solutions to some PDEs of mathematical physics*, Int. J. Math. Math. Sci. (2004), no. 21–24, 1105–1120.

[9] Oberguggenberger M., *Multiplication of distributions and applications to partial differential equations*, Pitman Research Notes in Mathematics Series 259, Longman Scientific & Technical, Harlow, 1992.

[10] Perthame B., *Kinetic formulation of conservation laws*, Oxford Lecture Series in Mathematics and its Applications 21, Oxford University Press, Oxford, 2002.

[11] Rubio J.E., *The global control of shock waves*, Nonlinear Theory of Generalized Functions (Vienna, 1997), Chapman & Hall/CRC Res. Notes Math. 401, Chapman & Hall/CRC, Boca Raton, FL, 1999, pp. 355–367.

[12] Rudin W., *Functional analysis*, McGraw-Hill Series in Higher Mathematics, McGraw-Hill Book Co., New York, 1973.

[13] Schwartz L., *Théorie des distributions*, Publications de l'Institut de Mathématique de l'Université de Strasbourg, No. IX–X, Nouvelle édition, entiérement corrigée, refondue et augmentée, Hermann, Paris, 1966.

[14] Shelkovich V.M. *New versions of the Colombeau algebras*, Math. Nachr. **278** (2005), no. 11, 1318–1340.

[15] Villarreal F., *Colombeau's theory and shock wave solutions for systems of PDEs*, Electron. J. Differential Equations 2000, no. 21, 17 pp.

[16] Ziemer W.P., *Weakly differentiable functions. Sobolev spaces and functions of bounded variation*, Graduate Texts in Mathematics 120, Springer, New York, 1989.