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Linear forms and axioms of choice

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Abstract: We work in set-theory without choice **ZF**. Given a commutative field \mathbb{K} , we consider the statement **D**(\mathbb{K}): “On every non null \mathbb{K} -vector space there exists a non-null linear form.” We investigate various statements which are equivalent to **D**(\mathbb{K}) in **ZF**. Denoting by \mathbb{Z}_2 the two-element field, we deduce that **D**(\mathbb{Z}_2) implies the axiom of choice for pairs. We also deduce that **D**(\mathbb{Q}) implies the axiom of choice for linearly ordered sets isomorphic with \mathbb{Z} .

Keywords: Axiom of Choice, axiom of finite choice, bases in a vector space, linear forms

AMS Subject Classification: Primary 03E25; Secondary 15A03

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