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Regular methods of summability in some locally convex spaces

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Abstract: Suppose that X is a Fréchet space, $\langle a_{ij} \rangle$ is a regular method of summability and (x_i) is a bounded sequence in X. We prove that there exists a subsequence (y_i) of (x_i) such that: either (a) all the subsequences of (y_i) are summable to a common limit with respect to $\langle a_{ij} \rangle$; or (b) no subsequence of (y_i) is summable with respect to $\langle a_{ij} \rangle$. This result generalizes the Erdös-Magidor theorem which refers to summability of bounded sequences in Banach spaces. We also show that two analogous results for some ω_1-locally convex spaces are consistent to ZFC.

Keywords: Fréchet space, regular method of summability, summable sequence, Galvin-Prikry theorem, Erdös-Magidor theorem

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