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Symmetric difference on orthomodular lattices and Z_2-valued states

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Abstract: The investigation of orthocomplemented lattices with a symmetric difference initiated the following question: Which orthomodular lattice can be embedded in an orthomodular lattice that allows for a symmetric difference? In this paper we present a necessary condition for such an embedding to exist. The condition is expressed in terms of Z_2-valued states and enables one, as a consequence, to clarify the situation in the important case of the lattice of projections in a Hilbert space.

Keywords: orthomodular lattice, quantum logic, symmetric difference, Boolean algebra, group-valued state
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