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Strict topologies and Banach-Steinhaus type theorems

Abstract: Let X be a completely regular Hausdorff space, E a real Banach space, and let $C_b(X, E)$ be the space of all E-valued bounded continuous functions on X. We study linear operators from $C_b(X, E)$ endowed with the strict topologies β_z ($z = \sigma, \tau, \infty, g$) to a real Banach space $(Y, \|\cdot\|_Y)$. In particular, we derive Banach-Steinhaus type theorems for $(\beta_z, \|\cdot\|_Y)$ continuous linear operators from $C_b(X, E)$ to Y. Moreover, we study σ-additive and τ-additive operators from $C_b(X, E)$ to Y.

Keywords: vector-valued continuous functions, strict topologies, locally solid topologies, Dini-topologies, strong Mackey space, σ-additive operators, τ-additive operators

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References

[AB]   Aliprantis C.D., Burkinshaw O., *Positive Operators*, Academic Press, New York, 1985.

[F]   Fontenot R.A., *Strict topologies for vector-valued functions*, Canad. J. Math. **26** (1974), 841–853.

[K]   Khurana S.S., *Topologies on spaces of vector-valued continuous functions*, Trans. Amer. Math. Soc. **241** (1978), 195–211.

[KO$_1$]   Khurana S.S., Othman S.I., *Convex compactnes property in some spaces of measures*, Math. Ann. **279** (1987), 345–348.

[KO$_2$]   Khurana S.S., Othman S.I., *Grothendieck measures*, J. London Math. Soc. (2) **39** (1989), 481–486.

[KO$_3$]   Khurana S.S., Othman S.I., *Completeness and sequential completeness in certain spaces of measures*, Math. Slovaca **45** (1995), no. 2, 163–170.

[KV]   Khurana S.S., Vielma J., *Weak sequential convergence and weak compactness in spaces of vector-valued continuous functions*, J. Math. Anal. Appl. **195** (1995), 251–260.

[NR]   Nowak M., Rzepka A., *Locally solid topologies on spaces of vector-valued continuous functions*, Comment. Math. Univ. Carolinae **43** (2002), no. 3, 473–483.

[SZ]   Schaefer H., Zhang X.-D., *On the Vitali-Hahn-Saks theorem*, Oper. Theory Adv. Appl., 75, Birkhäuser, Basel, 1995, pp. 289–297.