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Enveloping algebras of Malcev algebras

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Abstract: We first discuss the construction by Pérez-Izquierdo and Shestakov of universal nonassociative enveloping algebras of Malcev algebras. We then describe recent results on explicit structure constants for the universal enveloping algebras (both nonassociative and alternative) of the 4-dimensional solvable Malcev algebra and the 5-dimensional nilpotent Malcev algebra. We include a proof (due to Shestakov) that the universal alternative enveloping algebra of the real 7-dimensional simple Malcev algebra is isomorphic to the 8-dimensional division algebra of real octonions. We conclude with some brief remarks on tangent algebras of analytic Bol loops and monoassociative loops.

Keywords: Malcev algebras, universal enveloping algebras, universal alternative envelopes, differential operators, Bol algebras, analytic loops

AMS Subject Classification: Primary 17D10; Secondary 17B35, 17D05, 17D15, 20N05

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