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Octonionic Cayley spinors and E_6

Comment.Math.Univ.Carolin. 51,2 (2010) 193 –207.

Abstract: Attempts to extend our previous work using the octonions to describe fundamental particles lead naturally to the consideration of a particular real, noncompact form of the exceptional Lie group E_6, and of its subgroups. We are therefore led to a description of E_6 in terms of 3×3 octonionic matrices, generalizing previous results in the 2×2 case. Our treatment naturally includes a description of several important subgroups of E_6, notably G_2, F_4, and (the double cover of) $SO(9,1)$. An interpretation of the actions of these groups on the squares of 3-component *Cayley spinors* is suggested.

Keywords: octonions, E_6, exceptional Lie groups, Dirac equation
AMS Subject Classification: 17C90, 17A35, 22E70

References

[1] Baez J. C., *The octonions*, Bull. Amer. Math. Soc. **39** (2002), 145–205.
[2] Cartan É., *Le principe de dualité et la théorie des groupes simple et semi-simples*, Bull. Sci. Math. **49** (1925), 361–374.
[3] Conway J.H., Smith D.A., *On Quaternions and Octonions*, A.K. Peters, Natick, MA, 2003.
[4] Dray T., Manogue C.A., *The exceptional Jordan eigenvalue problem*, Internat. J. Theoret. Phys. **38** (1999), 2901–2916; (math-ph/9910004).
[5] Dray T., Manogue C.A., *Quaternionic spin*, in Clifford Algebras and their Applications in Mathematical Physics, (R. Abłamowicz and B. Fauser, eds.), Birkhäuser, Boston, 2000, pp. 29–46; (hep-th/9910010).
[6] Fairlie D.B., Corrigan E., *Private communication*, 1986.
[7] Freudenthal H., *Lie groups in the foundations of geometry*, Adv. Math. **1** (1964), 145–190.
[8] Harvey F.R., *Spinors and Calibrations*, Academic Press, Boston, 1990.
[9] Jacobson N., *Some Groups of Transformations defined by Jordan Algebras, II*, J. Reine Angew. Math. **204** (1960), 74–98.
[10] Manogue C.A., Dray T., *Dimensional reduction*, Mod. Phys. Lett. **A14** (1999), 99–103; (hep-th/9807044).
[11] Manogue C.A., Dray T., *Octonionic Möbius transformations*, Int. J. Mod. Phys. **A14** (1999), 1243–1255; (math-ph/9905024).
[12] Manogue C.A., Dray T., *Octonions, E_6, and particle physics*, in Proceedings of QUANTUM (York, 2008), J. Phys.: Conference Series (JPCS), to appear.
[13] Manogue C.A., Schray J., *Finite Lorentz transformations, automorphisms, and division algebras*, J. Math. Phys. **34** (1993), 3746–3767; (hep-th/9302044).
[14] Paige L.J., *Jordan algebras*, in Studies in Modern Algebra (A.A. Albert, ed.), Prentice Hall, Englewood Cliffs, NJ, 1963, pp. 144–186.
[15] Ramond P., *Introduction to exceptional Lie groups and algebras*, Caltech preprint CALT-68-577, 1976.
[16] Schafer R.D., *An Introduction to Nonassociative Algebras*, Academic Press, New York, 1966 (reprinted by Dover Publications, 1995).
[17] Schray J., *Octonions and supersymmetry*, PhD thesis, Oregon State University, 1994.
[18] Schray J., *The general classical solution of the superparticle*, Class. Quant. Grav. **13** (1996), 27–38; (hep-th/9407045).
[19] Sudbery A., *Division algebras, (pseudo)orthogonal groups and spinors*, J. Phys. **A17** (1984), 939–955.
[20] Wangberg A., *The structure of E_6*, PhD thesis, Oregon State University, 2007, (arXiv:0711.3447).
[21] Wangberg A., Dray T., *Visualizing Lie subalgebras using root and weight diagrams*, Loci **2**, February 2009; (mathdl.maa.org/mathDL/23/?pa=content&sa=viewDocument&nodeId=3287).