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Right division in Moufang loops

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Abstract: If $(G, \cdot)$ is a group, and the operation $(*)$ is defined by $x * y = x \cdot y^{-1}$ then by direct verification $(G, *)$ is a quasigroup which satisfies the identity $(x * y) * (z * y) = x * z$. Conversely, if one starts with a quasigroup satisfying the latter identity the group $(G, \cdot)$ can be constructed, so that in effect $(G, \cdot)$ is determined by its right division operation. Here the analogous situation is examined for a Moufang loop. Subtleties arise which are not present in the group case since there is a choice of defining identities and the identities produced by replacing loop multiplication by right division give identities in which loop inverses appear. However, it is possible with further work to obtain an identity in terms of $(*)$ alone. The construction of the Moufang loop from a quasigroup satisfying this identity is significantly more difficult than in the group case, and it was first carried out using the software Prover9. Subsequently a purely algebraic proof of the construction was obtained.

Keywords: Moufang loop, Prover9
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