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Closure-preserving covers in function spaces

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Abstract: It is shown that if $C_p(X)$ admits a closure-preserving cover by closed σ-compact sets then X is finite. If X is compact and $C_p(X)$ has a closure-preserving cover by separable subspaces then X is metrizable. We also prove that if $C_p(X,[0,1])$ has a closure-preserving cover by compact sets, then X is discrete.

Keywords: closure-preserving covers, function spaces, compact spaces, pointwise convergence topology, topological game, winning strategy
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References

[1] Arkhangelskii A.V., *Topological function spaces*, Mathematics and its Applications (Soviet Series), **78**, Kluwer Academic Publishers Group, Dordrecht, 1992.
[2] Junnila H.J.K., *Metacompactness, paracompactness, and interior-preserving open covers*, Trans. Amer. Math. Soc. **249** (1979), no. 2, 373–385.
[3] Engelking R., *General Topology*, Heldermann, Berlin, 1989.
[4] Potoczny H.B., *Closure-preserving families of compact sets*, General Topology and Appl. **3** (1973), 243–248.
[5] Potoczny H.B., Junnila H.J.K., *Closure-preserving families and metacompactness*, Proc. Amer. Math. Soc. **53** (1975), no. 2, 523–529.
[6] Rogers C.A., Jayne J.E., *K-analytic Sets*, Rogers C.A., Jayne J.E. *et al.*, "Analytic Sets", Academic Press, London, 1980, pp. 2–181.
[7] Shakmatov D.B., Tkachuk V.V., *When is the space $C_p(X)$ σ-countably compact?*, Vestnik Moskov. Univ. Mat. **41** (1986), no. 1, 73–75.
[8] Telgársky R., *Spaces defined by topological games*, Fund. Math. **88** (1975), no. 3, 193–223.
[9] Tkachuk V.V., *The spaces $C_p(X)$: decomposition into a countable union of bounded subspaces and completeness properties*, Topology Appl. **22** (1986), no. 3, 241–253.
[10] Tkachuk V.V., *The decomposition of $C_p(X)$ into a countable union of subspaces with "good" properties implies "good" properties of $C_p(X)$*, Trans. Moscow Math. Soc. **55** (1994), 239–248.