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On Kantorovich's result on the symmetry of Dini derivatives

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Abstract: For $f : (a, b) \to \mathbb{R}$, let A_f be the set of points at which f is Lipschitz from the left but not from the right. L.V. Kantorovich (1932) proved that, if f is continuous, then A_f is a "(k_d)-reducible set". The proofs of L. Zajíček (1981) and B.S. Thomson (1985) give that A_f is a σ-strongly right porous set for an arbitrary f. We discuss connections between these two results. The main motivation for the present note was the observation that Kantorovich's result implies the existence of a σ-strongly right porous set $A \subset (a, b)$ for which no continuous f with $A \subset A_f$ exists. Using Thomson's proof, we prove that such continuous f (resp. an arbitrary f) exists if and only if there exist strongly right porous sets A_n such that $A_n \nearrow A$. This characterization improves both results mentioned above.

Keywords: Dini derivative, one-sided Lipschitzness, σ-porous set, strong right porosity, abstract porosity
AMS Subject Classification: 26A27, 28A05

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