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On evolutionary Navier-Stokes-Fourier type systems in three spatial dimensions

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Abstract: In this paper, we establish the large-data and long-time existence of a suitable weak solution to an initial and boundary value problem driven by a system of partial differential equations consisting of the Navier-Stokes equations with the viscosity ν polynomially increasing with a scalar quantity k that evolves according to an evolutionary convection diffusion equation with the right hand side $\nu(k)|\mathbf{D}(\vec{v})|^2$ that is merely L^1-integrable over space and time. We also formulate a conjecture concerning regularity of such a solution.

Keywords: large data existence, suitable weak solution, Navier-Stokes-Fourier equations, incompressible fluid, the viscosity increasing with a scalar quantity, regularity, turbulent kinetic energy model
AMS Subject Classification: 35Q30, 35Q35, 76F60

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