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Near heaps

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Abstract: On any involuted semigroup $(S, \cdot, ')$, define the ternary operation $[abc] := a \cdot b' \cdot c$ for all $a, b, c \in S$. The resulting ternary algebra $(S, [\,])$ satisfies the para-associativity law $[[abc]de] = [a[dcb]e] = [ab[cde]]$, which defines the variety of semiheaps. Important subvarieties include generalised heaps, which arise from inverse semigroups, and heaps, which arise from groups. We consider the intermediate variety of near heaps, defined by the additional laws $[aaa] = a$ and $[aab] = [baa]$. Every Clifford semigroup is a near heap when viewed as a semiheap, and we show that the Clifford semigroup operations are determined by the semiheap operation. We show that near heaps are exactly strong semilattices of heaps, parallelling a known result for Clifford semigroups. We characterise those near heaps which arise directly from Clifford semigroups, and show that all near heaps are embeddable in such examples, extending known results of this kind relating heaps to groups, generalised heaps to inverse semigroups, and general semiheaps to involuted semigroups.

Keywords: Clifford semigroups, semiheaps, generalised heaps, heaps
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