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*Curvature bounds for neighborhoods of self-similar sets*

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**Abstract:** In some recent work, fractal curvatures $C_k^f(F)$ and fractal curvature measures $C_k^f(F,\cdot)$, $k = 0,\dots,d$, have been determined for all self-similar sets $F$ in $\mathbb{R}^d$, for which the parallel neighborhoods satisfy a certain regularity condition and a certain rather technical curvature bound. The regularity condition is conjectured to be always satisfied, while the curvature bound has recently been shown to fail in some concrete examples. As a step towards a better understanding of its meaning, we discuss several equivalent formulations of the curvature bound condition and also a very natural technically simpler condition which turns out to be stronger. These reformulations show that the validity of this condition does not depend on the choice of the open set and the constant $R$ appearing in the condition and allow to discuss some concrete examples of self-similar sets. In particular, it is shown that the class of sets satisfying the curvature bound condition is strictly larger than the class of sets satisfying the assumption of polyconvexity used in earlier results.

**Keywords:** self-similar set, parallel set, curvature measures, fractal curvatures, Minkowski content, Minkowski dimension, regularity condition, curvature bound condition
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