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Closed discrete subsets of separable spaces and relative versions of normality, countable paracompactness and property (a)

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Abstract: In this paper we show that a separable space cannot include closed discrete subsets which have the cardinality of the continuum and satisfy relative versions of any of the following topological properties: normality, countable paracompactness and property (a). It follows that it is consistent that closed discrete subsets of a separable space X which are also relatively normal (relatively countably paracompact, relatively (a)) in X are necessarily countable. There are, however, consistent examples of separable spaces with uncountable closed discrete subsets under the described relative topological requirements, and therefore the existence of such uncountable sets is undecidable within ZFC. We also investigate what are the outcomes of considering the set-theoretical hypothesis "$2^\omega < 2^{\omega_1}$" within our discussion and conclude by giving some notes and posing some questions.

Keywords: relative normality, relative countable paracompactness, relative property (a), closed discrete subsets, separable spaces
AMS Subject Classification: Primary 54D20, 54A25, 54A35; Secondary 54B05, 54D45, 03E55

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