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Uncountably many solutions of a system of third order nonlinear differential equations

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Abstract: In this paper, we aim to study the global solvability of the following system of third order nonlinear neutral delay differential equations

$$\begin{aligned}&\frac{d}{dt}\Big\{r_i(t)\frac{d}{dt}\Big[\lambda_i(t)\frac{d}{dt}\Big(x_i(t)-f_i(t,x_1(t-\sigma_{i1}),x_2(t-\sigma_{i2}),x_3(t-\sigma_{i3}))\Big)\Big]\Big\}\\&\qquad+\frac{d}{dt}\Big[r_i(t)\frac{d}{dt}g_i(t,x_1(p_{i1}(t)),x_2(p_{i2}(t)),x_3(p_{i3}(t)))\Big]\\&\qquad+\frac{d}{dt}h_i(t,x_1(q_{i1}(t)),x_2(q_{i2}(t)),x_3(q_{i3}(t)))\\&=l_i(t,x_1(\eta_{i1}(t)),x_2(\eta_{i2}(t)),x_3(\eta_{i3}(t))),\quad t\ge t_0,\quad i\in\{1,2,3\}\end{aligned}$$

in the following bounded closed and convex set

$$\Omega(a,b)=\Big\{x(t)=\big(x_1(t),x_2(t),x_3(t)\big)\in C([t_0,+\infty),\mathbb{R}^3): a(t)\le x_i(t)\le b(t),\qquad \forall\, t\ge t_0, i\in\{1,2,3\}\Big\},$$

where $\sigma_{ij}>0$, $r_i,\lambda_i,a,b\in C([t_0,+\infty),\mathbb{R}^+)$, $f_i,g_i,h_i,l_i\in C([t_0,+\infty)\times\mathbb{R}^3,\mathbb{R})$, $p_{ij},q_{ij},\eta_{ij}\in C([t_0,+\infty),\mathbb{R})$ for $i,j\in\{1,2,3\}$. By applying the Krasnoselskii fixed point theorem, the Schauder fixed point theorem, the Sadovskii fixed point theorem and the Banach contraction principle, four existence results of uncountably many bounded positive solutions of the system are established.

Keywords: system of third order nonlinear neutral delay differential equations, contraction mapping, completely continuous mapping, condensing mapping, uncountably many bounded positive solutions

AMS Subject Classification: 34K15, 34C10

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