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Disconnectedness properties of hyperspaces

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Abstract: Let X be a Hausdorff space and let $\mathcal{H}$ be one of the hyperspaces $CL(X)$, $\mathcal{K}(X)$, $\mathcal{F}(X)$ or $\mathcal{F}_n(X)$ (n a positive integer) with the Vietoris topology. We study the following disconnectedness properties for $\mathcal{H}$: extremal disconnectedness, being a F'-space, P-space or weak P-space and hereditary disconnectedness. Our main result states: if X is Hausdorff and $F \subset X$ is a closed subset such that (a) both F and $X - F$ are totally disconnected, (b) the quotient X/F is hereditarily disconnected, then $\mathcal{K}(X)$ is hereditarily disconnected. We also show an example proving that this result cannot be reversed.

Keywords: hyperspaces, Vietoris topology, F'-space, P-space, hereditarily disconnected
AMS Subject Classification: 54B20, 54G05, 54G10, 54G12, 54G20

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