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Manifold-valued generalized functions in full Colombeau spaces

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Abstract: We introduce the notion of generalized function taking values in a smooth manifold into the setting of full Colombeau algebras. After deriving a number of characterization results we also introduce a corresponding concept of generalized vector bundle homomorphisms and, based on this, provide a definition of tangent map for such generalized functions.

Keywords: algebras of generalized functions, manifold-valued generalized functions, full Colombeau algebras
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References

[1] Abraham R., Marsden J.E., Ratiu T., *Manifolds, Tensor Analysis, and Applications*, Applied Mathematical Sciences, 75, Springer, New York, 1988.
[2] Clarke C.J.S., Vickers J.A., Wilson J.P., *Generalized functions and distributional curvature of cosmic strings*, Classical Quantum Gravity **13** (1996), no. 9, 2485–2498.
[3] Colombeau J.-F., *New Generalized Functions and Multiplication of Distributions*, North-Holland Mathematics Studies, 84, North-Holland, Amsterdam, 1984.
[4] Colombeau J.-F., *Elementary Introduction to New Generalized Functions*, North-Holland Mathematics Studies, 113, North-Holland, Amsterdam, 1985.
[5] Colombeau J.-F., Meril A., *Generalized functions and multiplication of distributions on C^∞ manifolds*, J. Math. Anal. Appl. **186** (1994), no. 2, 357–364.
[6] Dapić N., Pilipović S., *Microlocal analysis of Colombeau's generalized functions on a manifold*, Indag. Math. (N.S.) **7** (1996), no. 3, 293–309.
[7] de Roever J.W., Damsma,M., *Colombeau algebras on a C^∞-manifold*, Indag. Math. (N.S.) **2** (1991), no. 3, 341–358.
[8] Erlacher E., Grosser M., *Inversion of a "discontinuous coordinate transformation" in general relativity*, Applicable Analysis, to appear.
[9] Grosser M., Farkas E., Kunzinger M., Steinbauer R., *On the foundations of nonlinear generalized functions I and II*, Mem. Amer. Math. Soc. **153** (2001), no. 729.
[10] Grosser M., Kunzinger M., Oberguggenberger M., Steinbauer R., *Geometric Theory of Generalized Functions with Applications to General Relativity*, Kluwer, Dordrecht, 2001.
[11] Grosser M., Kunzinger M., Steinbauer R., Vickers J.A., *A global theory of algebras of generalized functions*, Adv. Math. **166** (2002), no. 1, 50–72.
[12] Grosser M., Kunzinger M., Steinbauer R., Vickers J.A., *A global theory of algebras of generalized functions II: tensor distributions*, submitted, `http://arxiv.org/abs/0902.1865`
[13] Jelinek J., *An intrinsic definition of the Colombeau generalized functions*, Comment. Math. Univ. Carolin. **45** (1999), no. 1, 71–95.
[14] Jelinek J., *Colombeau product of currents*, Comment. Math. Univ. Carolin. **46** (2005), no. 3, 437–462.
[15] Kriegl A., Michor P., *The Convenient Setting of Global Analysis*, Mathematical Surveys and Monographs, 53, American Mathematical Society, Providence, 1997.
[16] Kunzinger M., *Generalized functions valued in a smooth manifold*, Monatsh. Math. **137** (2002), 31–49.
[17] Kunzinger M., Steinbauer R., *Foundations of a nonlinear distributional geometry*, Acta Appl. Math. **71** (2002), no. 2, 179–206.
[18] Kunzinger, M., Steinbauer, R., Vickers, J.A., *Intrinsic characterization of manifold-valued generalized functions*, Proc. London Math. Soc. **87** (2003), no. 2, 451–470.
[19] Kunzinger M., Steinbauer R., Vickers J.A., *Sheaves of nonlinear generalized functions and manifold-valued distributions*, Trans. Amer. Math. Soc. **361** (2009), 5177–5192.
[20] Marsden J.E., *Generalized Hamiltonian mechanics: A mathematical exposition of non-smooth dynamical systems and classical Hamiltonian mechanics*, Arch. Rational Mech. Anal. **28** (1967/1968), 323–361.

[21] Nedeljkov M., Pilipović S., Scarpalezos D., *The Linear Theory of Colombeau Generalized Functions*, Pitman Research Notes in Mathematics, 385, Longman, Harlow, 1998.

[22] Nigsch E., *Approximation properties of local smoothing kernels*, Integral Transform. Spec. Funct., to appear.

[23] Oberguggenberger M., *Multiplication of distributions and applications to partial differential equations*, Pitman Research Notes in Mathematics Series, 259, Longman Scientific & Technical, Harlow, 1992.

[24] O'Neill B., *Semi-Riemannian Geometry. With Applications to Relativity*, Pure and Applied Mathematics, 103, Academic Press, New York, 1983.

[25] Parker P.E., *Distributional geometry*, J. Math. Phys. **20** (1979), no. 7, 1423–1426.

[26] Steinbauer R., Vickers J.A., *The use of generalized functions and distributions in general relativity*, Classical Quantum Gravity **23** (2006), no. 10, R91–R114.

[27] Vernaeve H., *Isomorphisms in algebras of Colombeau generalized functions*, Monatsh. Math. **162** (2011), 225–237.

[28] Vickers J., Wilson J., *A nonlinear theory of tensor distributions*, ESI-Preprint **566** (`http://www.esi.ac.at/ESI-Preprints.html`), 1998.

[29] Vickers J.A., Wilson J.P., *Invariance of the distributional curvature of the cone under smooth diffeomorphisms*, Classical Quantum Gravity **16** (1999), no. 2, 579–588.