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Maximal free sequences in a Boolean algebra

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Abstract: We study free sequences and related notions on Boolean algebras. A *free sequence* on a BA A is a sequence $\langle a_\xi : \xi < \alpha\rangle$ of elements of A, with α an ordinal, such that for all $F, G \in [\alpha]^{<\omega}$ with $F < G$ we have $\prod_{\xi\in F} a_\xi \cdot \prod_{\xi\in G} -a_\xi \neq 0$. A free sequence of length α exists iff the Stone space $\mathrm{Ult}(A)$ has a free sequence of length α in the topological sense. A free sequence is *maximal* iff it cannot be extended at the end to a longer free sequence. The main notions studied here are the spectrum function

$$\mathfrak{f}_{\mathrm{sp}}(A) = \{|\alpha| : A \text{ has an infinite maximal free sequence of length } \alpha\}$$

and the associated min-max function

$$\mathfrak{f}(A) = \min(\mathfrak{f}_{\mathrm{sp}}(A)).$$

Among the results are: for infinite cardinals $\kappa \leq \lambda$ there is a BA A such that $\mathfrak{f}_{\mathrm{sp}}(A)$ is the collection of all cardinals μ with $\kappa \leq \mu \leq \lambda$; maximal free sequences in A give rise to towers in homomorphic images of A; a characterization of $\mathfrak{f}_{\mathrm{sp}}(A)$ for A a weak product of free BAs; $\mathfrak{p}(A), \pi\chi_{\mathrm{inf}}(A) \leq \mathfrak{f}(A)$ for A atomless; a characterization of infinite BAs whose Stone spaces have an infinite maximal free sequence; a generalization of free sequences to free chains over any linearly ordered set, and the relationship of this generalization to the supremum of lengths of homomorphic images.

Keywords: free sequences, cardinal functions, Boolean algebras
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