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Berezin transform for non-scalar holomorphic discrete series

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Abstract: Let $M = G/K$ be a Hermitian symmetric space of the non-compact type and let π be a discrete series representation of G which is holomorphically induced from a unitary irreducible representation ρ of K. In the paper [B. Cahen, *Berezin quantization for holomorphic discrete series representations: the non-scalar case*, Beiträge Algebra Geom., DOI 10.1007/s13366-011-0066-2], we have introduced a notion of complex-valued Berezin symbol for an operator acting on the space of π. Here we study the corresponding Berezin transform and we show that it can be extended to a large class of symbols. As an application, we construct a Stratonovich-Weyl correspondence associated with π.

Keywords: Berezin quantization, Berezin symbol, Stratonovich-Weyl correspondence, discrete series representation, Hermitian symmetric space of the non-compact type, semi-simple non-compact Lie group, coherent states, reproducing kernel, adjoint orbit
AMS Subject Classification: 22E46, 32M10, 32M15, 81S10

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