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*On special partitions of Dedekind- and Russell-sets*

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**Abstract:** A *Russell set* is a set which can be written as the union of a countable pairwise disjoint set of pairs no infinite subset of which has a choice function and a *Russell cardinal* is the cardinal number of a Russell set. We show that if a Russell cardinal  $a$  has a ternary partition (see Section 1, Definition 2) then the Russell cardinal  $a + 2$  fails to have such a partition. In fact, we prove that if a ZF-model contains a Russell set, then it contains Russell sets with ternary partitions as well as Russell sets without ternary partitions. We then consider generalizations of this result.

**Keywords:** Axiom of Choice, Dedekind sets, Russell sets, generalizations of Russell sets, odd sized partitions, permutation models

**AMS Subject Classification:** 03E10, 03E25, 03E35, 05A18

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