Svatopluk Krýsl
Symplectic Killing spinors

Abstract: Let (M, ω) be a symplectic manifold admitting a metaplectic structure (a symplectic analogue of the Riemannian spin structure) and a torsion-free symplectic connection ∇. Symplectic Killing spinor fields for this structure are sections of the symplectic spinor bundle satisfying a certain first order partial differential equation and they are the main object of this paper. We derive a necessary condition which has to be satisfied by a symplectic Killing spinor field. Using this condition one may easily compute the symplectic Killing spinor fields for the standard symplectic vector spaces and the round sphere S^2 equipped with the volume form of the round metric.

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