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H-closed extensions with countable remainder

Comment.Math.Univ.Carolin. 53,1 (2012) 123 –137.

Abstract: This paper investigates necessary and sufficient conditions for a space to have an H-closed extension with countable remainder. For countable spaces we are able to give two characterizations of those spaces admitting an H-closed extension with countable remainder. The general case is more difficult, however, we arrive at a necessary condition — a generalization of Čech completeness, and several sufficient conditions for a space to have an H-closed extension with countable remainder. In particular, using the notation of Császár, we show that a space X is a Čech g-space if and only if X is G_δ in σX or equivalently if EX is Čech complete. An example of a space which is a Čech f-space but not a Čech g-space is given answering a couple of questions of Császár. We show that if X is a Čech g-space and $R(EX)$, the residue of EX, is Lindelöf, then X has an H-closed extension with countable remainder. Finally, we investigate some natural generalizations of the residue to the class of all Hausdorff spaces.

Keywords: Čech complete, H-closed, extension
AMS Subject Classification: 54A25, 54D35, 54D40

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