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Distinct equilateral triangle dissections of convex regions

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Abstract: We define a proper triangulation to be a dissection of an integer sided equilateral triangle into smaller, integer sided equilateral triangles such that no point is the vertex of more than three of the smaller triangles. In this paper we establish necessary and sufficient conditions for a proper triangulation of a convex region to exist. Moreover we establish precisely when at least two such equilateral triangle dissections exist. We also provide necessary and sufficient conditions for some convex regions with up to four sides to have either one, or at least two, proper triangulations when an internal triangle is specified.

Keywords: equilateral triangle dissection, latin trade

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