

Stephen Gagola III

Nonsplitting F -quasigroups

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Abstract: T. Kepka, M.K. Kinyon and J.D. Phillips [*The structure of F -quasigroups*, J. Algebra **317** (2007), no. 2, 435–461] developed a connection between F -quasigroups and NK-loops. Since NK-loops are contained in the variety generated by groups and commutative Moufang loops, a question that arises is whether or not there exists a nonsplit NK-loop and likewise a nonsplit F -quasigroup. Here we prove that there do indeed exist nonsplit F -quasigroups and show that there are exactly four corresponding nonsplit NK-loops of minimal order 3^6 .

Keywords: F -quasigroup, NK-loop, Moufang loop

AMS Subject Classification: Primary 20E10; Secondary 20N05

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