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*Pseudoautomorphisms of Bruck loops and their generalizations*

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**Abstract:** We show that in a weak commutative inverse property loop, such as a Bruck loop, if  $\alpha$  is a right [left] pseudoautomorphism with companion  $c$ , then  $c [c^2]$  must lie in the left nucleus. In particular, for any such loop with trivial left nucleus, every right pseudoautomorphism is an automorphism and if the squaring map is a permutation, then every left pseudoautomorphism is an automorphism as well. We also show that every pseudoautomorphism of a commutative inverse property loop is an automorphism, generalizing a well-known result of Bruck.

**Keywords:** pseudoautomorphism, Bruck loop, weak commutative inverse property

**AMS Subject Classification:** 20N05

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