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Solution of distributive-like quasigroup functional equations

Abstract: We are investigating quasigroup functional equation classification up to parastrophic equivalence [Sokhatsky F.M., *On classification of functional equations on quasigroups*, Ukrainian Math. J. **56** (2004), no. 4, 1259–1266 (in Ukrainian)]. If functional equations are parastrophically equivalent, then their functional variables can be renamed in such a way that the obtained equations are equivalent, i.e., their solution sets are equal. There exist five classes of generalized distributive-like quasigroup functional equations up to parastrophic equivalence [Sokhatsky F.M., *On classification of distributive-like functional equations*, Book of Abstracts of the 8th International Algebraic Conference in Ukraine, July 5–12 (2011), Lugansk, Ukraine, p. 79]. In the article, we find the solution sets of four generalized distributive-like quasigroup functional equations of different classes. In consequence, we solve one of the equations on topological quasigroup operations, defined on arbitrary topological space as well as on the space of real numbers with the natural topology. The fifth class contains the generalized left distributivity functional equation. V.D. Belousov [*Some remarks on the functional equation of generalized distributivity*, Aequationes Math. **1** (1968), no. 1–2, 54–65] described only a subset of its solution set. The set of all solutions still remains an open problem in the quasigroup theory and in the functional equation theory.

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