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Fixed points of periodic and firmly lipschitzian mappings in Banach spaces

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Abstract: W.A. Kirk in 1971 showed that if $T\colon C \to C$, where C is a closed and convex subset of a Banach space, is n-periodic and uniformly k-lipschitzian mapping with $k < k_0(n)$, then T has a fixed point. This result implies estimates of $k_0(n)$ for natural $n \geq 2$ for the general class of k-lipschitzian mappings. In these cases, $k_0(n)$ are less than or equal to 2. Using very simple method we extend this and later results for a certain subclass of the family of k-lipschitzian mappings. In the paper we show that $k_0(3) > 2$ in any Banach space. We also show that $\mathrm{Fix}(T)$ is a Hölder continuous retract of C.

Keywords: lipschitzian mapping, firmly lipschitzian mapping, n-periodic mapping, fixed point, retractions
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