Ali Rezaei Aliabad, Mehdi Badie

Fixed-place ideals in commutative rings

Comment.Math.Univ.Carolin. 54,1 (2013) 53 –68.

Abstract: Let I be a semi-prime ideal. Then $P_\circ \in \mathrm{Min}(I)$ is called irredundant with respect to I if $I \neq \bigcap_{P_\circ \neq P \in \mathrm{Min}(I)} P$. If I is the intersection of all irredundant ideals with respect to I, it is called a fixed-place ideal. If there are no irredundant ideals with respect to I, it is called an anti fixed-place ideal. We show that each semi-prime ideal has a unique representation as an intersection of a fixed-place ideal and an anti fixed-place ideal. We say the point $p \in \beta X$ is a fixed-place point if $O^p(X)$ is a fixed-place ideal. In this situation the fixed-place rank of p, denoted by $\text{FP-rank}_X(p)$, is defined as the cardinal of the set of all irredundant prime ideals with respect to $O^p(X)$. Let p be a fixed-place point, it is shown that $\text{FP-rank}_X(p) = \eta$ if and only if there is a family $\{Y_\alpha\}_{\alpha\in A}$ of cozero sets of X such that: 1- $|A| = \eta$, 2- $p \in \mathrm{cl}_{\beta X} Y_\alpha$ for each $\alpha \in A$, 3- $p \notin \mathrm{cl}_{\beta X}(Y_\alpha \cap Y_\beta)$ if $\alpha \neq \beta$ and 4- η is the greatest cardinal with the above properties. In this case p is an F-point with respect to Y_α for any $\alpha \in A$.

Keywords: ring of continuous functions, fixed-place, anti fixed-place, irredundant, semi-prime, annihilator, affiliated prime, fixed-place rank, Zariski topology
AMS Subject Classification: Primary 13Axx; Secondary 54C40

References

[1] Aliabad A.R., *z°-ideals in $C(X)$*, Ph.D. Thesis, Chamran University of Ahvaz, Iran.
[2] Aliabad A.R., *Connections between $C(X)$ and $C(Y)$, where Y is a subspace of X*, Abstracts of International Conference on Applicable General Topology, August 12–18, 2001, Hacettepe University, Ankara, Turkey.
[3] Aliabad A.R., *Pasting topological spaces at one point*, Czechoslovak Math. J. **56** (131) (2006), 1193–1206.
[4] Aliabad A.R., Badie M., *Connection between $C(X)$ and $C(Y)$, where Y is subspace of X*, Bull. Iranian Math. Soc. **37** (2011), no. 4, 109–126.
[5] Engelking R., *General Topology*, PWN-Polish Scientific Publishing, Warsaw, 1977.
[6] Gillman L., Jerison M., *Rings of Continuous Functions*, Van Nostrand Reinhold, New York, 1960.
[7] Goodearl K.R., Warfield R.B., Jr. *Introduction to Noncommutative Noetherian Rings*, Cambridge University Press, Cambridge, 1989.
[8] Henriksen M., Larson S., Martinez J., Woods R.G., *Lattice-ordered algebras that are subdirect product of valuation domains*, Trans. Amer. Math. Soc. **345** (1994), 195–221.
[9] Henriksen M., Wilson R.G., *Almost discrete SV-space*, Topology Appl. **46** (1992), 89–97.
[10] Larson S., *f-Rings in which every maximal ideal contains finitely many minimal prime ideals*, Comm. Algebra **25** (1997), no. 12, 3859–3888.
[11] Larson S., *Constructing rings of continuous functions in which there are many maximal ideals with nontrivial rank*, Comm. Algebra **31** (2003), 2183–2206.
[12] Underwood D.H., *On some Uniqueness questions in primary representations of ideals*, Kyoto Math. J. **35** (1969), 69–94.
[13] Willard S., *General Topology*, Addison Wesley, Reading, Mass., 1970.