Józef Banaś, Afif Ben Amar*

Measures of noncompactness in locally convex spaces and fixed point theory for the sum of two operators on unbounded convex sets

Comment.Math.Univ.Carolin. 54,1 (2013) 21 –40.

Abstract: In this paper we prove a collection of new fixed point theorems for operators of the form $T+S$ on an unbounded closed convex subset of a Hausdorff topological vector space (E,Γ). We also introduce the concept of demi-τ-compact operator and τ-semi-closed operator at the origin. Moreover, a series of new fixed point theorems of Krasnosel'skii type is proved for the sum $T+S$ of two operators, where T is τ-sequentially continuous and τ-compact while S is τ-sequentially continuous (and Φ_τ-condensing, Φ_τ-nonexpansive or nonlinear contraction or nonexpansive). The main condition in our results is formulated in terms of axiomatic τ-measures of noncompactness. Apart from that we show the applicability of some our results to the theory of integral equations in the Lebesgue space.

Keywords: τ-measure of noncompactness, τ-sequential continuity, Φ_τ-condensing operator, Φ_τ-nonexpansive operator, nonlinear contraction, fixed point theorem, demi-τ-compactness, operator τ-semi-closed at origin, Lebesgue space, integral equation
AMS Subject Classification: 47H10

References

[1] Appell J., De Pascale E., *Su alcuni parametri connessi con la misura di non compattezza di Hausdorff in spazi funzioni misurabili*, Boll. Un. Mat. Ital. B (6) **3** (1984), 497–515.
[2] Appell J., Zabrejko P.P., *Nonlinear Superposition Operators*, Cambridge University Press, Cambridge, 1990.
[3] Arino O., Gautier S., Penot J.P., *A fixed point theorem for sequentially continuous mappings with application to ordinary differential equations*, Funkcial. Ekvac. **27** (1984), no. 3, 273–279.
[4] Banaś J., *Demicontinuity and weak sequential continuity of operators in the Lebesgue space*, Proceedings of the 1st Polish Symposium on Nonlinear Analysis, 1997, pp. 124–129.
[5] Banaś J., *Applications of measures of weak noncompcatness and some classes of operators in the theory of functional equations in the Lebesgue space*, Nonlinear Anal. **30** (1997), no. 6, 3283–3293.
[6] Barroso C.S., *Krasnoselskii's fixed point theorem for weakly continuous maps*, Nonlinear Anal. **55** (2003), 25–31.
[7] Barroso C.S., Teixeira E.V., *A topological and geometric approach to fixed point results for sum of operators and applications*, Nonlinear Anal. **60** (2005), no. 4, 625–660.
[8] Barroso C.S., Kalenda O.F.K., Rebouąs M.P., *Optimal approximate fixed point results in locally convex spaces*, J. Math. Anal. Appl. **401** (2013), no. 1, 1–8.
[9] Ben Amar A., Jeribi A., Mnif M., *On a generalization of the Schauder and Krasnosel'skii fixed point theorems on Dunford-Pettis space and applications*, Math. Methods Appl. Sci. **28** (2006), 1737–1756.
[10] Ben Amar A., Jeribi A., Mnif M., *Some fixed point theorems and application to biological model*, Numer. Funct. Anal. Optim. **29** (2008), no. 1–2, 1-23.
[11] Ben Amar A., Mnif M., *Leray-Schauder alternatives for weakly sequentially continuous mappings and application to transport equation*, Math. Methods Appl. Sci. **33** (2010), no. 1, 80–90.
[12] Ben Amar A., Xu S., *Measures of weak noncompactness and fixed point theory for 1-set weakly contractive operators on unbounded domains*, Anal. Theory Appl. **27** (2011), no. 3, 224–238.
[13] Boyd D.W., Wong J.S.W., *On nonlinear contractions*, Proc. Amer. Math. Soc. **20** (1969), 458–464.
[14] Burton T.A., *A fixed point theorem of Krasnosel'skii*, Appl. Math. Lett. **11** (1998), 85–88.
[15] Day M.M., *Normed Linear Spaces*, Academic Press, New York, 1962.

[16] De Blasi F.S., *On a property of the unit sphere in Banach space*, Bull. Math. Soc. Sci. Math. R.S. Roumanie **21** (1977), 259–262.

[17] Dunford N., Pettis B.J., *Linear operators on summable functions]/*, Trans. Amer. Math. Soc. **47** (1940), 323–392.

[18] Dunford N., Schwartz J.T., *Linear Operators, Part I*, Interscience, Leyden, 1963.

[19] Edwards R.E., *Functional Analysis, Theory and Applications*, Holt, Reinhard and Winston, New York, 1965.

[20] Garcia-Falset J., *Existence of fixed points and measures of weak noncompactness*, Nonlinear Anal. **71** (2009), 2625–2633.

[21] Krasnosel'skii M.A., *On the continuity of the operator* $Fu(x) = f(x, u(x))$, Dokl. Akad. Nauk SSSR **77** (1951), 185–188 (in Russian).

[22] Krasnosel'skii M.A., *Two remarks on the method of successive approximation*, Uspehi Mat. Nauk **10** (1955), 123–127 (in Russian).

[23] Krasnosel'skii M.A., Zabrejko P.P., Pustyl'nik J.I., Sobolevskii P.J., *Integral Opertors in Spaces of Summable Functions*, Noordhoff, Leyden, 1976.

[24] Kubiaczyk I., *On a fixed point theorem for weakly sequentially continuous mappings*, Discuss. Math. Differential Incl. **15** (1995), 15–20.

[25] O'Regan D., *Fixed-point theory for weakly sequentially continuous mappings*, Math. Comput. Modelling **27** (1998), no. 5, 1–14.

[26] O'Regan D., Taoudi M.A., *Fixed point theorems for the sum of two weakly sequentially continuous mappins*, Nonlinear Anal. **73** (2010), 283–289.

[27] V.I. Shragin, *On the weak continuity of the Nemytskii operator*, Uchen. Zap. Mosk. Obl. Ped. Inst. **57** (1957), 73–79.

[28] Taoudi M.A., *Krasnosel'skii type fixed point theorems under weak topology features*, Nonlin. Anal. **72** (2010), no. 1, 478–482.

[29] Zabrejko P.P., Koshelev A.I., Krasnosel'skii M.A., Mikhlin S.G., Rakovshchik L.S., Stecenko V.J., *Integral Equations*, Noordhoff, Leyden, 1975.