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Frame monomorphisms and a feature of the l-group of Baire functions on a topological space

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Abstract: The "kernel functor" $W \xrightarrow{k}$ LFrm from the category W of archimedean lattice-ordered groups with distinguished weak unit onto LFrm, of Lindelöf completely regular frames, preserves and reflects monics. In W, monics are one-to-one, but not necessarily so in LFrm. An embedding $\varphi \in W$ for which $k\varphi$ is one-to-one is termed kernel-injective, or KI; these are the topic of this paper. The situation is contrasted with kernel-surjective and -preserving (KS and KP). The W-objects every embedding of which is KI are characterized; this identifies the LFrm-objects out of which every monic is one-to-one. The issue of when a W-map $G \xrightarrow{\varphi} \cdot$ is KI is reduced to when a related epicompletion of G is KI. The poset $EC(G)$ of epicompletions of G is reasonably well-understood; in particular, it has the functorial maximum denoted βG, and for $G = C(X)$, the Baire functions $B(X) \in EC(C(X))$. The main theorem is: $E \in EC(C(X))$ is KI iff $B(X) \overset{*}{\leq} E \overset{*}{\leq} \beta C(X)$ in the order of $EC(C(X))$. This further identifies in a concrete way many LFrm-monics which are/are not one-to-one.

Keywords: Baire functions, archimedean lattice-ordered group, Lindelöf frame, monomorphism
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