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On character of points in the Higson corona of a metric space

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Abstract: We prove that for an unbounded metric space X, the minimal character $\mathsf{m}\chi(\check{X})$ of a point of the Higson corona $\check{X}$ of X is equal to $\mathfrak{u}$ if X has asymptotically isolated balls and to $\max\{\mathfrak{u},\mathfrak{d}\}$ otherwise. This implies that under $\mathfrak{u}<\mathfrak{d}$ a metric space X of bounded geometry is coarsely equivalent to the Cantor macro-cube $2^{<\mathbb{N}}$ if and only if $\dim(\check{X})=0$ and $\mathsf{m}\chi(\check{X})=\mathfrak{d}$. This contrasts with a result of Protasov saying that under CH the coronas of any two asymptotically zero-dimensional unbounded metric separable spaces are homeomorphic.

Keywords: Higson corona, character of a point, ultrafilter number, dominating number
AMS Subject Classification: 03E17, 03E35, 03E50, 54D35, 54E35, 54F45

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