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Productivity of the Zariski topology on groups

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Abstract: This paper investigates the productivity of the Zariski topology \mathfrak{Z}_G of a group G . If $\mathcal{G} = \{G_i \mid i \in I\}$ is a family of groups, and $G = \prod_{i \in I} G_i$ is their direct product, we prove that $\mathfrak{Z}_G \subseteq \prod_{i \in I} \mathfrak{Z}_{G_i}$. This inclusion can be proper in general, and we describe the doubletons $\mathcal{G} = \{G_1, G_2\}$ of abelian groups, for which the converse inclusion holds as well, i.e., $\mathfrak{Z}_G = \mathfrak{Z}_{G_1} \times \mathfrak{Z}_{G_2}$. If $e_2 \in G_2$ is the identity element of a group G_2 , we also describe the class Δ of groups G_2 such that $G_1 \times \{e_2\}$ is an elementary algebraic subset of $G_1 \times G_2$ for every group G_1 . We show among others, that Δ is stable under taking finite products and arbitrary powers and we describe the direct products that belong to Δ . In particular, Δ contains arbitrary direct products of free non-abelian groups.

Keywords: Zariski topology, (elementary, additively) algebraic subset, δ -word, universal word, verbal function, (semi) \mathfrak{Z} -productive pair of groups, direct product

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