Néjib Ben Salem, Walid Nefzi

Images of some functions and functional spaces under the Dunkl-Hermite semigroup

Comment.Math.Univ.Carolin. 54,3 (2013) 345 –365.

Abstract: We propose the study of some questions related to the Dunkl-Hermite semigroup. Essentially, we characterize the images of the Dunkl-Hermite-Sobolev space, $\mathcal{S}(\mathbb{R})$ and $L^p_\alpha(\mathbb{R})$, $1 < p < \infty$, under the Dunkl-Hermite semigroup. Also, we consider the image of the space of tempered distributions and we give Paley-Wiener type theorems for the transforms given by the Dunkl-Hermite semigroup.

Keywords: Dunkl-Hermite functions; Dunkl-Hermite semigroup; Dunkl-Hermite-Sobolev space
AMS Subject Classification: 42B25, 46E35, 47B38, 47D03

References

[1] Ben Salem N., Nefzi W., *Inversion of the Dunkl-Hermite semigroup*, Bull. Malays. Math. Sci. Soc. (2) **35** (2012), no. 2, 287–301.
[2] Ben Salem N., Samaali T., *Hilbert transforms associated with Dunkl-Hermite polynomials*, SIGMA Symmetry Integrability Geom. Methods Appl. **5** (2009), Paper 037.
[3] Dunkl C.F., *Integral kernels with reflection group invariance*, Canad. J. Math. **43** (1991), 1213–1227.
[4] Erdely A. et al., *Higher Transcendental Functions, vol 2*, McGraw-Hill, New-York, 1953.
[5] de Jeu M.F.E., *The Dunkl transform*, Invent. Math. **113** (1993), 147–162.
[6] Lebedev N.N., *Special Functions and their Applications*, translated by R.A. Silverman, Dover, New York, 1972.
[7] Maalaoui R., Trimèche K., *A family of generalized windowed transforms associated with the Dunkl operators on* $\mathbb{R}^d$, Integral Transforms Spec. Funct. **23** (2012), no. 3, 191–206.
[8] Radha R., Thangavelu S., *Holomorphic Sobolev spaces, Hermite and special Hermite semigroups and a Paley-Wiener theorem for the windowed Fourier transform*, J. Math. Anal. Appl. **354** (2009), 564–574.
[9] Radha R., Venku Naidu D., *Image of* $L^p(\mathbb{R}^n)$ *under the Hermite semigroup*. Int. J. Math. Math. Sci. (2008), Art. ID 287218, 13 pages.
[10] Rosenblum M., *Generalized Hermite polynomials and the Bose-like oscillator calculus*, in Operator theory: Advances and Applications, Vol. 73, Birkhäuser, Basel, 1994, pp. 369–396.
[11] Rösler M., *Generalized Hermite polynomials and the heat equation for Dunkl operators*, Comm. Math. Phys. **192** (1998), no. 3, 519–542.
[12] Rösler M., Voit M., *Markov Processes related with Dunkl operators*, Adv. Appl. Math. **21** (1998), 575–643.
[13] Trimèche K., *Paley-Wiener theorems for the Dunkl transform and Dunkl translation operators*, Integral Transforms Spec. Funct. **13** (2002), 17–38.
[14] Trimèche K., *Hypoelliptic Dunkl convolution equations in the space of distributions on* $\mathbb{R}^d$, J. Fourier Anal. Appl. **12** (2006), 517–542.