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Some remarks on distributive semilattices

Comment.Math.Univ.Carolin. 54,3 (2013) 407–428.

Abstract: In this paper we shall give a survey of the most important characterizations of the notion of distributivity in semilattices with greatest element and we will present some new ones through annihilators and relative maximal filters. We shall also simplify the topological representation for distributive semilattices given in Celani S.A., *Topological representation of distributive semilattices*, Sci. Math. Japonicae online **8** (2003), 41–51, and show that the meet-relations are closed under composition. So, we obtain that the *DS*-spaces with meet-relations is a category dual to the category of distributive semilattices with homomorphisms. These results complete the topological representation presented in Celani S.A., *Topological representation of distributive semilattices*, Sci. Math. Japonicae online **8** (2003), 41–51, without the use of ordered topological spaces. Finally, following the work of G. Bezhanishvili and R. Jansana in *Generalized Priestley quasi-orders*, Order **28** (2011), 201–220, we will prove a characterization of homomorphic images of a distributive semilattice A by means of family of closed subsets of the dual space endowed with a lower Vietoris topology.

Keywords: distributive semilattices; topological representation; meet-relations

AMS Subject Classification: Primary 03G10, 06A12; Secondary 06D50

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