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Smoothness properties of solutions to the nonlinear Stokes problem with nonautonomous potentials

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Abstract: We discuss regularity results concerning local minimizers $u : \mathbb{R}^n \supset \Omega \to \mathbb{R}^n$ of variational integrals like

$$\int_\Omega \{F(\cdot, \varepsilon(w)) - f \cdot w\}\, dx$$

defined on energy classes of solenoidal fields. For the potential F we assume a (p,q)-elliptic growth condition. In the situation without x-dependence it is known that minimizers are of class $C^{1,\alpha}$ on an open subset Ω_0 of Ω with full measure if $q < p\,\frac{n+2}{n}$ (for $n = 2$ we have $\Omega_0 = \Omega$). In this article we extend this to the case of nonautonomous integrands. Of course our result extends to weak solutions of the corresponding nonlinear Stokes type system.

Keywords: Stokes problem; generalized Newtonian fluids; regularity; nonautonomous functionals; slow flows

AMS Subject Classification: 76M30, 76D07, 49N60, 35J50

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